Elementary Möbius Geometry

Initialization Notebook

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Mathematica v. 11.3, v. 8.0.1

```
In[1]:=   Date[][[1 ;; 3]]
```

Contents

This notebook loads the packages needed to initialize the interactive work with the notebooks about Möbius geometry.

Section 1 is used to initialize this work.

Section 2 gives useful informations about the Möbius group and its Lie algebra. The Lie algebra programs are contained in the package liealgsh.m which must be imported in the relevant Sections only.

First initialize this notebook as described in Section 1. Then open the wanted notebook by clicking on the wanted of the following links:

emg1.nb Elementary Möbius Geometry I, Points and Spheres

emg2.nb Elementary Möbius Geometry II, Circles

emg3.nb Elementary Möbius Geometry III, Pairs of Subspheres in S^3

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1. Initialization

Before starting first time to work interactively with the notebooks emgi.nb, i= 0,1,.., of elementary Möbius geometry, read this section carefully and make the necessary preparation. Later it suffices to call the menu item "Evaluation. Evaluate Initialization Cells".

1.1. The needed packages

For working with the notebooks on Möbius geometry you need the packages euvecv2.m, tensalgv3.m, neuvecv2.m, mspher.m, mcirc.m, moeb3.m, eudiffgeov4.m. The package lielgsh.m about Lie algebras is needed in the relevant sections of the notebooks only.
You may download these and other packages from my homepage. In the downloaded file elmoeb.zip you find all the packages mentioned above, together with the notebooks and other files on

Möbius geometry. Before initializing the notebook copy all these files into your working directory, e.g. the directory "E:\\mymath\\emg" for Windows, or "~/mymath/emdg" for Linux. Insert the path of your working directory into the command of the next cell corresponding to your operating system:

For Windows:

In[2]:=
```
SetDirectory["H:\\mymath\\diffgeo\\mdg\\elmoebius"];
```

For Linux:

```
SetDirectory["~/mymath/diffgeo/mdg/elmoebius"];
```

Now give the cell corresponding to your operating system the properties "Cell Evaluatable" and "Initialization cell" (Menu Cell/Cell Properties), and deactivate these properties for the cell corresponding to the other operating system. If this is done, save the notebook. Next time you may start the notebook directly with the evaluation of the initialization, as follows:

1.2. The Initialization

To work interactively with the notebooks,

Activite Evaluation/Evaluate Initialization Cells from the menu.

The first command loads the packages mentioned above:

In[25]:=
```
Needs["moeb3`"]
```

In this notebook we shall consider the 3-dimensional Möbius space as the elliptical quadric, the 3-dimensional sphere, in the 4-dimensional real projective space. Therefore we set the dimension dim and the index ind of the vector space as follows:

In[4]:=
```
    dim::usage="dim is the dimension of the vector space under
 consideration; it must be set within the Global Context.";
```

```
? dim
```

In[5]:=
```
    dim =5;
```

In[6]:=
```
ind ::usage="ind is the index of the pseudo-Euclidean vector space under consideration; it mus
```

In[7]:=
```
ind =1;
```

$Assumptions

We always assume that the global variables appearing in the next command are real numbers:

In[8]:=
```
$Assumptions= s∈Reals&&t∈Reals&&u∈Reals&&v∈Reals&&w∈Reals&&x∈Reals&&y∈Reals&&z∈Reals;
```

In[ ]:=
```
$Assumptions
```

1.3. List of Symbols and their Usages

In this Chapter one finds tables of the symbols introduced in the imported packages and in the Global Context.
To get the usages click on the name! If this does not work correctly, enable Dynamic Updating in the Evaluation Menu.

1.3.1. Symbols in the Package tensalgv3.m

1.3.2. Symbols in the Package euvecv2.m

1.3.3. Symbols in the Package neuvecv2.m

1.3.4. Symbols in the Package moeb3.m

1.3.5. Symbols in the Package eudiffgeov4.m

1.3.6. Symbols in the Package mcirc.m

1.3.7. Symbols in the Package mspher.m

1.3.8. Symbols in the Global Context

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2. The Möbius Group

As remarked in the Introduction, the Möbius group consists of all time-orientation preserving elements of the pseudo-orthogonal group, denoted by $O(1, 4)^+$, see [OS3]. Proposition 2.7.1. In this Chapter we give two matrix representations of its Lie algebra and consider how some well known geometric groups are realized as subgroups of the Möbius group.

We use the basic operation "commute" of Lie algebras; thus we load

In[ ]:= **Needs["liealgsh`"]**

Symbols in the Package liealgsh.m

2.1. The Lie Algebra o(1,4) of the Möbius Group

2.2. Lines. The Conformal Representation of the Translation Group and the Dilation Group

2.3. Circles. The Conformal Representation of the Rotation Group

2.4. The Conformal and isometry Groups of the Space Forms

In the following subsections we find the subgroups of the Möbius group leaving the embedded space forms invariant. More exactly speaking, we find their Lie algebras as subalgebras of o(1,4).

2.4.1. The Conformal Euclidean Group

2.4.2. The isometry Groups of the 3-Sphere and the Hyperbolic Space

2.5. A Moving Frame on the Sphere Manifold: pstg5frame

References

Homepage

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