

Loxodromes

by Rolf Sulanke, October 26, 2009

The aim of this notebook is to show that the loxodroms besides of the circles are the curves of constant conformal curvature in the Möbius plane. We calculate the conformal natural parameter of a plane curve and the moving Frenet Frame of the loxodromes. The relation between Euclidean and Möbius invariants are deduced. On the way we define and apply basic concepts of Euclidean differential geometry of plane curves.

■ Copyright

■ Keywords

Plane Curves, spherical curves, arclength, curvature, conformal curvature, natural conformal parameter, spirals, loxodromes

■ Initialization

The needed packages

Initialization

1. List of Symbols and their Usages

2. The Loxodromes

■ 2.1. Logarithmic Spirals as Orbits of 1-parameter Groups of Conformal Transformations

■ 2.2. Loxodromes as Images of the Spirals under Inverse Stereographic Projection

3. Euclidean Curve Theory

In this section we construct Modules for the Euclidean curve theory. For detailed presentation of Euclidean differential geometry with *Mathematica* see the book [2] of Alfred Gray

3.1 Curves in the Euclidean Plane

4. Frenet Formulas in the Möbius Plane S^2

- 4.1 Conformal Embedding of the Euclidean Space
- 4.2 Loxodromes as Conformal Embeddings of the Logarithmic Spirals
- 4.3 Möbius Geometry of Curves in S^2 Applied to the Loxodromes
- 4.4 Example: The Natural parameter is proportional to the group parameter of a loxodrome.
- 4.5 The Conformal Curvature Expressed by the Euclidean Curvature of a Plane Curve
- 4.6. The Spiral Group as a Subgroup of the Pseudo-Orthogonal Group $O(1,3)$

References

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