Euclidean Curve Theory

by Rolf Sulanke, June 17, 2009

Summary

In this notebook we develop *Mathematica* tools for the Euclidean differential geometry of curves. We construct Modules for the calculation of all Euclidean invariants like arclength, curvatures, and Frenet formulas in the plane, the 3-space, and in n-dimensional Euclidean spaces. As an application we show that the curves of constant curvatures in the 4-dimensional Euclidean space are isogonal trajectories of certain circular tori and visualize them by sterepographical projection. A short presentation of Euclidean curve theory as it is used in present notebook is given in my paper [ECG] which may be down-loaded from my homepage.

Initialization

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1. List of Symbols and their Usages

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2. Curves in the Euclidean plane

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3. Curves in the Euclidean Space

Now we consider curves in the Eucliden space. Our aim is to find expressions for the curvature and the torsion of the curve F(t), where t is an arbitrary parameter and F(t) the radius vector of the curve.

4. The Frenet Formulas

- **4.1 Frenet Formulas for Plane Curves**
- **4.2 frenet Formulas for Space Curves**
- **4.3 Frenet Frames for Curves in the Euclidean Space of n Dimensions**
- **4.4 The Curvatures of Curves in the Euclidean Space of n Dimensions**

5. Curves of Constant Curvatures and 1-Parameter Motion Groups

- **5.1 Screw Motions in the Euclidean 3-Space**
- **5.2 A Degenerated Case: m[0,b]**
- **5.3 Curves of Constant Curvatures in the Euclidean 4-Space**
- **5.4 The Shape of the Orbits of Rank 4**
- **5.5 Isogonality**

References

[G06] Alfred Gray, Simon Salamon, Elsa Abbena. Modern Differential Geometry of Curves and Surfaces with *Mathematica*.
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[G94] Alfred Gray. Differentialgeometrie. Klassische Theorie in moderner Darstellung. (Übersetzung aus dem Amerikanischen H. Gollek).
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[ECG] Rolf Sulanke. The Fundamental Theorem for Curves in the n-Dimensional Euclidean Space. 2009. On my homepage.

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Homepage

http://www-irm.mathematik.hu-berlin.de/~sulanke