1-Semiquasihomogeneous Singularities of Hypersurfaces in Characteristic 2

Abstract:
In arbitrary characteristic different from 2, the singularities with semiquasihomogeneous equations characterized by the condition to have Saito-invariant 1 are the "classical" quasihomogeneous ones, known over the field of complex numbers as simple elliptic singularities (Saito, [10]). Here we find them in characteristic 2 as well: In odd dimensions and for weights $E_6$ and $E_8$ non-quasihomogeneous equations appear.

0. The problem
$k$ denotes an algebraically closed field. Let $X$ be a finite set of indeterminates $x$ equipped with positive weights $w(x) \in \mathbb{Q}$ and $f \in k[[X]]$ be a formal power series consisting of monomials of weight $\geq 1$ such that $f_1 := \text{sum of terms of total degree 1}$ defines an isolated singularity (i.e. the partial derivatives generate an ideal which is primary for the maximal ideal in $k[[X]]/(f)$). Then we associate to $f$ the "Saito-invariant" $s := |X| - 2 \sum_{x \in X} w(x)$. We say "$f$ is $s$-semiquasihomogeneous" (or short: "$s$-sqh") with respect to the given weights. For $f = f_1$, $f$ is said to be "$1$-quasihomogeneous". The case of $s < 1$ gives the rational double points (the simple singularities or, equivalently, the absolutely isolated Cohen Macaulay double points, cf. [3], [6], [4]). Here the "boundary case" of $s = 1$ is considered, which corresponds in the complex-analytic case to the simple elliptic singularities ([10]). Note however, that for $\text{char } k = 2$ not all of those singularities arise from dimension 2, so here they better will be referred to only as 1-semiquasihomogeneous. As for the simple singularities, the case of characteristic 2 is most complicated in the sense of stable equivalence for different dimensions. From the point of representations (considering the Auslander-Reiten quiver of maximal Cohen-Macaulay modules over the local ring of the singularity), the usual Knörrer-periodicity has to be replaced by Solberg’s periodicity (taking dimensions $mod$ 2), and the results of Kahn (cf. [5]) may apply at least to some of the singularities found here.

1. The quasihomogeneous case
Write $X = \{X_0, \ldots, X_n\}$ and $w(X_i) = w_i$. We always assume $w_i \leq \frac{1}{2}$; this is no loss of generality (cf. e.g. [6]). Let $k[X]_1$ denote the polynomials which are sum of monomials of weight 1 (set of "quasihomogeneous polynomials" with respect to the given weights). Then we have the following

Cancellation property: Let $f, g \in k[X]_1$ define isolated singularities, and let $q_1, q_2 \in k[Y]$ be nondegenerate quadratic forms in a finite set $Y$ of new variables of weight $\frac{1}{2}$. Suppose $f + q_1$ can be transformed into $g + q_2$ by an automorphism
Φ of \( k[X, Y] \) preserving the grading. Then there exists an automorphism \( Ψ \) of \( k[X] \) which preserves the grading and such that \( f = g \circ Ψ \).

This is a consequence of the following (cf. [6])

**Proposition (Saito, Knop):** Choose \( f ∈ k[[X]] \) defining an isolated singularity.

(i) If \( Y ⊆ X \), then one of the following is satisfied:
   (a) There exists \( X^α ∈ supp(f) \) such that \( X^α ∈ k[Y] \), or
   (b) There exists an injective map \( ϕ : Y → X − Y \) and a map \( ψ : Y → RV^Y \) such that \( Y^{ϕ(y)} \cdot y \cdot ϕ(y) ∈ supp(f) \) for every \( y ∈ Y \).

(ii) Assume \( f \) is quasihomogeneous of degree 1. Then up to an automorphism of \( k[X] \) which preserves the grading, \( f = f_1 + \sum_{x ∈ A} xϕ(x) \), where \( A = \{x ∈ X, w(x) > \frac{1}{2} \} \) and \( ϕ : A ↪ X − A \) is an injection, \( f_1 ∈ k[X − (A ∪ ϕ(A))] \). Now, choose all \( w_i ≤ \frac{1}{2} \) and denote \( Q := \{x ∈ X | w(x) = \frac{1}{2}\}, R := X − Q = \{x ∈ X | w(x) < \frac{1}{2}\} \).

Up to a graded automorphism, \( f \) is of the following form:
   (a) \( f = f_1 + q, f_1 ∈ k[R], \) and \( q ∈ k[Q] \) a nondegenerate quadratic form.
   (b) \( char k = 2, \) and there exists \( x_0 ∈ Q \) such that \( f = f_1 + f_2 \cdot x_0 + x_0^2 + q \), where \( q ∈ k[Q − \{x_0\}] \) (q nondegenerate quadratic form), \( f_i ∈ k[R] \) for \( i = 1, 2 \).

We deduce a

**Proof of the cancellation property:**

Let \( f + q_1 = (g + q_2) \circ Φ \). In case of part (ii) (a) of the preceding proposition, we may assume \( X = R, \) i.e. \( f, g ∈ (X_0, . . . , X_n)^2, w(X_i) < \frac{1}{2}, \) thus \( Φ(X_i) ∈ k[X], \) and after a linear change of coordinates in \( Y, \) \( Φ(Y_i) = Y_i. \)

Now let \( char k = 2 \) and suppose \( f \) has the form (ii) of (b), \( f + q_1 = f_1 + f_2x_0 + X_0^2 + q, \) where \( f_i ∈ k[X_1, . . . , X_n] \) and \( q ∈ k[Y] \) is a nondegenerate quadratic form. We may assume \( g + q_2 = g_1 + g_2x_0 + X_0^2 + q, \) and also \( g_i, f_i ∈ k[Y], |Y| = m \) even and \( q = Y_1Y_2 + . . . + Y_{m−1}Y_m \) (classification of quadratic forms in characteristic 2). Then, if \( f = g ◦ Φ, \) \( Φ \) graded. We obtain \( Φ(R) ⊆ k[R], R = \{X_1, . . . , X_n\}, Φ \)

induces a linear transformation in the variables \( \{X_0\} \cup Y \ mod(X_1, . . . , X_n)^2, \) fixing \( X_0^2 + q(Y) \ mod(X_1, . . . , X_n)^2. \) Thus we may assume \( Φ(X_0) = X_0 + φ_0, \) \( Φ(Y_i) = Y_i + φ_i, \) \( φ_1 ∈ k[X] \) of weight \( \frac{1}{2}. \) But \( q \) is nondegenerate, thus \( φ_1 = . . . = φ_m = 0. \)

**Definition:** Choose \( f ∈ k[[X]] \) and \( g ∈ k[[X']] \).

(i) \( f, g \) are said to be right equivalent if \( X = X' \) and there exists an automorphism \( Φ \) of \( k[[X]] \) such that \( f = g \circ Φ. \) In this case, we write \( f \sim g \) (without loss of generality, \( Φ \) can be chosen homogeneous of degree 0 if \( f ∈ k[X]_1, \) \( g ∈ k[X]_1 \) for a fixed weight \( w \)).

\(^1\) tacitly assumed to be of degree 0
(ii) Assume there exist nondegenerate quadratic forms $q \in k[Z]$, $q' \in k[Z']$ respectively in finite sets $Z$, resp. $Z'$ of new variables such that $f + q \sim g + q'$. Then $f, g$ are said to be stable-equivalent$^2$. We write $f \sim g$. The polynomials $f + q$, $g + q'$ respectively will be referred to as "quadratic suspensions" of $f, g$ respectively.

Thus, the above cancellation property says: If $f, g$ (as above) have the same number of variables and $f \sim g$, then $f \sim g$

If $f \sim g$, then $s(f) = s(g)$, and always $0 \leq s(f) < |X|$. The classes of $f$ having $s(f) < 1$ are precisely the quasihomogeneous forms of the simple singularities $ADE$ (cf. [6], [4]); their behavior under the canonical local resolution is studied in [7].

For the 1-qh polynomials we have the following

**Theorem:** Let $f \in k[X]$ be a polynomial defining an isolated singularity such that $f$ is quasihomogeneous for some weight $w$ with $s = 1$.

Then $w$ is (up to permutation) one of the weights

$$\tilde{E}_0 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \ldots, \frac{1}{2}), \quad \tilde{E}_7 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \ldots, \frac{1}{2}), \quad \tilde{E}_8 = (\frac{1}{3}, \frac{1}{6}, \frac{1}{2}, \ldots, \frac{1}{2})$$

and $f$ is stable-equivalent with one of the following polynomials ($t \in k$ denotes a parameter):

**Case A:** $\text{char}(k) \neq 2$

- $\tilde{E}_0$: $f = X_0(X_1 - X_0)(X_1 - tX_0) - X_0X_2^2, \quad t \neq 0, 1$
- $\tilde{E}_7$: $f = X_0X_1(X_1 - X_0)(X_1 - tX_0), \quad t \neq 0, 1$
- $\tilde{E}_8$: $f = X_0(X_0 - X_1^2)(X_0 - tX_1^2), \quad t \neq 0, 1$

**Case B:** $\text{char}(k) = 2$

1. $n$ odd

- $\tilde{E}_0(0)$: $X_0^3 + X_1^2X_2 + X_1X_2^2 + X_3^2$
- $\tilde{E}_0(t)$: $X_0^3 + tX_1^2 + X_1^2X_2 + X_0X_1X_2 + X_3^2, \quad t \neq 0$
- $\tilde{E}_7(t)$: $X_0X_1(X_1 + X_0)(X_1 + tX_0), \quad t \neq 0, 1$
- $\tilde{E}_8(t)$: $X_0(X_0 + X_1^2)(X_0 + tX_1^2), \quad t \neq 0, 1$

$^2$Note, the condition implies that the total number of variables has to be the correct one.
2. \( n \) even

\[ \begin{align*}
\hat{E}_0(0) & : \quad X_0^3 + X_1^2X_2 + X_1X_2^2 \\
\hat{E}_0(t) & : \quad X_0^3 + tX_2^3 + X_1^2X_2 + X_0X_1X_2, \quad t \neq 0 \\
\hat{E}_{7,1}(t) & : \quad X_0^3 + X_0X_1^2 + X_1X_2(tX_1 + X_2) \\
\hat{E}_{7,2}(t) & : \quad X_0^2 + X_0X_1X_2 + X_1X_2(tX_1 + X_2)^2, \quad t \neq 0 \\
\hat{E}_0(t) & : \quad X_0^2 + X_0X_1X_2 + X_1(X_1 + X_2^2)(X_1 + tX_2^2), \quad t \neq 0
\end{align*} \]

Proof: To start with, we need the following

Lemma: With the previous notations, assume \( s = |X| - 2\sum_{x \in X} w(x) = 1 \), i.e.

\[ \sum_{x \in R} w(x) = \frac{1}{2}(|R| - 1) \]

and such that there exists a polynomial \( f \in k[X]_1 \) with an isolated singularity. Then

(i) \( |R| \neq 0, 1 \)

(ii) \( S := \{ x \in X \mid \frac{1}{3} < w(x) < \frac{1}{2} \} = \emptyset \)

(iii) \(|R| \leq 3 \) with equality at most if \( w_0 = w_1 = w_2 = \frac{1}{3} \) (up to permutation of indices of \( X_i \)).

(iv) If \(|R| = 2 \), then \( w_0 = w_1 = \frac{1}{4} \), or \( w_0 = \frac{1}{3} \), \( w_1 = \frac{1}{6} \) (again, indices may permute).

Proof of the Lemma: (i) is an obvious consequence of \( s = 1 \).

To show (ii), (iii), apply (i) in the above proposition: Choose maps \( \varphi, \psi \) with the property (b) and obtain:

\[ \frac{1}{2}(|R| - 1) = \sum_{x \in R} w(x) = \]

\[ = \sum_{x \in S} w(x) + \sum_{x \in S} w(\varphi(x)) + \sum_{x \in R \setminus (S \cup \varphi(S))} w(x) \]

\[ = \sum_{x \in S} w(x) + \sum_{x \in S} (1 - w(x) - w(S^x)) + \sum_{x \in R \setminus (S \cup \varphi(S))} w(x) \]

\[ = \sum_{x \in S} (1 - w(S^x)) + \sum_{x \in R \setminus (S \cup \varphi(S))} w(x) \leq \frac{2}{3}|S| + \frac{1}{3}|R - S \cup \varphi(S)| \]
(note that $S^{\psi(x)} \cdot x \cdot \varphi(x) \in \text{supp}(f)$ for all $x$, i.e. $w(S^{\psi(x)}) + w(x) + w(\varphi(x)) = 1$; also, $\varphi(S) \subseteq R$). Thus

$$\frac{1}{2}(|R| - 1) \leq \frac{1}{3}(2|S| + |R - (S \cup \varphi(S))|) = \frac{1}{3}|R|.$$ 

To prove (iv), we may assume $w_0 + w_1 = \frac{1}{3}$, $w_i = \frac{1}{3}$ for $i > 1$, i.e. for $w_0 = w_1$ we are done. Assume $w_0 > w_1$, then $\frac{1}{4} < w_0 < \frac{1}{2}$. If $X_0^3 \notin \text{supp}(f)$, then no power of $X^3$ is in $\text{supp}(f)$, and (i) (a) in the Proposition implies (using $Y = X_0$) that one of the monomials $X_0^{\alpha+1}X_i$ ($\alpha \in \mathbb{N}, i \in \{1, \ldots, n\}$) is in $\text{supp}(f)$. This implies $w_0 = \frac{1}{2x}$ or $w_0 = \frac{1}{2(\alpha+1)}$ (contradiction, since $\alpha \in \mathbb{N}$). Thus $X_0^3 \in \text{supp}(f)$, i.e. $w_0 = \frac{1}{3}$, $w_1 = \frac{1}{3}$.

Now, a detailed case by case analysis gives the

**Proof of the Theorem:**

Choose e.g. the case of $\hat{E}_0$ in even dimension, i.e. here without loss of generality in dimension 2. Then in coordinates $(x_0 : x_1 : x_2)$, the corresponding equation $f = 0$ defines a smooth curve $C$ of degree 3 in the projective plane. We obtain the above normal form after a linear change of coordinates. In char $k = 2$ we have two cases: $\hat{E}_0(0)$ if the elliptic curve is supersingular, $\hat{E}_0(t)$, with $t \neq 0$ otherwise.

For the weights $\hat{E}_7$, $\hat{E}_8$, a geometric analysis of the relevant forms is necessary, giving different equations in even and odd dimensions for char $k = 2$.

We apply the proposition to obtain the list of equations; choose e.g. $f$ of weight $\hat{E}_7$, char $k = 2$:

We may assume $X = \{X_0, \ldots, X_n\}$ with

(a) $n = 1$, $w_0 = w_1 = \frac{1}{4}$, $f = f(X_0, X_1)$ homogeneous of degree 4 and defining an isolated singularity, i.e. $f$ with 4 different zeroes on $\mathbb{P}^1$.

(b) $n = 2$, $w_0 = \frac{1}{7}$, $w_1 = w_2 = \frac{1}{3}$, $f = x_2 + gx_0 + h$, $g \in k[X_1, X_2]$ homogeneous of degree 2, $h \in k[X_1, X_2]$ homogeneous of degree 4.

If (b1) $g = 0$, then coordinates can be chosen such that $X_1X_2^3 \notin \text{supp} h$, thus $V(X_1, X_0^2 + g(X_1, X_2)) \subseteq \text{sing}(f)$, i.e. the singular locus has positive dimension. Now assume (b2) $g = X_1^2$, then $f = X_0^2 + X_0X_1^2 + h(X_1, X_2)$. Write $h(X_1, X_2) = \sum_{n=0}^{1} h_n X_1^n X_2^{4-n}$. Then $f$ defines an isolated singularity iff $h_1 \neq 0$; we may assume $h_1 = 1$. A coordinate transformation $X_0 := X_0 + aX_1^2 + bX_1X_2 + cX_2^2$ brings $h$ into the form $h = X_1X_2^2(tX_1 + X_2)$.

The case (b3) $g = X_1X_2$ is done in a similar way.
Remark: Note that also in char $k = 2$, the equations for $E_6$ can be written in a form such that $E_6(0)$ and $E_6(t)$, $t \neq 0$ are in the same 1-parameter family: Take $n = 2$ and let $C(s)$ be the curve defined in the projective plane by

$$X_0^3 + X_1^3 + X_2^3 + sX_0X_1X_2 = 0$$

where $s \in k$. For $s^3 \neq 1$ this is an elliptic curve with absolute invariant $j = \frac{s^{12}}{(s^3 + 1)}$, and $E_6(0)$ is the cone over an elliptic curve with invariant 0, thus isomorphic to the cone over $C(0)$. For fixed $t \neq 0$, the equation $ts^{12} + s^6 + s^3 + 1 = 0$ has 12 different solutions $s$. We obtain several $C(s)$ with invariant $j = \frac{1}{7}$. Thus any 2-dimensional quasihomogeneous singularity of type $E_6$ is obtained as cone over some $C(s)$.

Corollary: Let $f \in k[X]$ be quasihomogeneous of some weight $w = w(f)$ and assume $s = s(f) \leq 1$. Then $w$ is uniquely determined up to permutation in the class of quasihomogeneous functions which are stable equivalent $f$. Especially, the number $s$ is well defined on the equivalence class.

Remark: In the case considered here, $w$ (up to permutation) and therefore $s(f)$ depends only on the complete local ring of the singularity. It is not known to the author, if this is generally so for $s(f) > 1$ (but it is always true for $k = \mathcal{O}$ by [10]).

2. Normal forms of semiquasihomogeneous functions

Now let $f = f_1 + f_{>1}$ be a formal power series which contains no monomials of weight $< 1$ with respect to the given weight $w$. Put $f_1 := \text{sum of terms of weight 1 in } f$ and assume $f_1$ defines an isolated singularity.

$f$ is said to be contact equivalent with a power series $g$, if the $k$-algebras $k[[X]]/(f)$ and $k[[X]]/(g)$ of formal power series are isomorphic.

The following result reduces the part $f_{>1}$ into a normal form without changing $f_1$ and the contact equivalence class of $f$. $T(f_1)$ denotes the "Tjurina-algebra",

$$T(f_1) := k[[X]]/(f_1, \frac{\partial f_1}{\partial X_0}, \ldots, \frac{\partial f_1}{\partial X_n})$$

We have $\text{dim}_k(T(f_1)) < \infty$.

Theorem: Let $(\bar{e}_1, \ldots, \bar{e}_s)$ denote any maximal linear independent set of classes in $T(f_1)$ of monomials $e_i$ having weight $> 1$ ("superdiagonal monomials"). Then $f = f_1 + f_{>1}$ is contact equivalent with $f_1 + c_1e_1 + \ldots + c_se_s$, $e_i \in k$.

Proof: Let $w = (\frac{m_0}{d}, \ldots, \frac{m_n}{d})$ with positive integers $m_i, d$. Denote $o_m(h)$ the total order of the initial term of a power series $h \in k[[X_0, \ldots, X_n]]$ with respect to $(m_0, \ldots, m_n)$.
If the classes of superdiagonal monomials \( \{ e_1, \ldots, e_s \} \) form a basis of the subspace generated by all superdiagonal monomials in the Tjurina algebra \( T(f_1) \), then the similar assertion is true for any fixed order \( d' \), i.e. let \( \{ e_{ij} \} \) be the subset of monomials such that \( o_m(e_{ij}) = d' \), then this is a basis for the subspace in \( T(f_1) \) generated by the classes of all monomials having \( o_m = d' \) (\( f_1 \) is homogeneous).

Obviously, an inductive convergence argument gives the result, if we show the following

**Lemma:** Let (after some permutation) \( e_1, \ldots, e_r \) be the monomials of order \( o_m(e_i) = d' > d \) in \( \{ e_1, \ldots, e_s \} \). Then \( f \) is contact equivalent with a series

\[
f_1 + f'_{>1} + \sum_{i=1}^r e_i + h,
\]

where \( f'_{>1} \) is the sum of terms of order \( o_m < d' \) in \( f_{>1} \), \( e_i \in k \) and \( h \in k[[X_0, \ldots, X_n]] \) has order \( o_m(h) > d' \).

(Note that the case is included, where \( \{ e_1, \ldots, e_r \} \) is the empty set.)

**Proof of the Lemma:** Choose \( e_i \in k \) such that

\[
g - \sum_{i=1}^r c_i e_i = q \cdot f_1 + \sum_{i=0}^n v_i \frac{\partial f_1}{\partial X_i}
\]

for some \( q, v_i \in k[[X_0, \ldots, X_n]] \) and \( g \) the sum of monomials of order \( d' \) in \( f_{>1} \).

Without loss of generality, \( q \) and \( v_i \) are quasihomogeneous for \( (m_0, \ldots, m_n) \) of order

\[
o_m(q) = d' - d =: \delta > 0
\]

\[
o_m(v_i) = d' - (d - m_i) = \delta + m_i > m_i,
\]

respectively. We obtain

\[
(*) \quad f_1 + f'_{>1} + \sum_{i=1}^r c_i e_i = (1 - q)(f_1 + f_{>1}) + qf_{>1} - \sum_{i=0}^n v_i \frac{\partial f_1}{\partial X_i} + p,
\]

where in the right hand term \( o_m(qf_{>1}) > d' \), \( v_i \frac{\partial f_1}{\partial X_i} \) is quasihomogeneous with \( o_m(v_i \frac{\partial f_1}{\partial X_i}) = d' \), and \( o_m(p) > d' \).

Assume without loss of generality \( m_0 \geq m_1 \geq \ldots \geq m_n \). Let \( X_i := X_i' - v_i(X_i) \), then \( o_m(v_i) > m_i = o_m(X_i) \) implies: The linear part of this coordinate transformation has a lower triangular matrix \( (a_{ij}) \) with \( a_{ii} = 1 \), and \( a_{ij} \neq 0 \) for \( i > j \) is possible only if \( m_i > m_j \). The above substitution sends

\[
(**) \quad f_1(X) \mapsto f_1(X') - \sum_{i=0}^r v_i(X') \frac{\partial f_1(X')}{\partial X_i'} + \text{terms in } X' \text{ of order } o_m > d'
\]
(if we take the same weights for the $X'$).
By ($\ast$), we have
\[
(**) \quad (1 - \eta(X)) f(X) \equiv f_1 + \sum_{i=0}^n v_i(X) \frac{\partial f_1}{\partial X_i} + (f_{>1}(X) + \sum_{i=0}^r c_i e_i(X))
\]

mod terms of order $o_m > d'$. If we apply (***) and remember $o_m(v_i) > d' - d$, the substitution above transforms the right hand side of (***) into
\[
f_1(X') + (f_{>1}(X') + \sum_{i=0}^r c_i e_i(X')) + h(X'),
\]
where $o_m(h) > d'$. This completes the proof.

Note that we do not need any assumption on char $k$. If char $k = 0$, by Euler’s formula we have $f_1 \in \left( \frac{\partial f_1}{\partial X_0}, \ldots, \frac{\partial f_1}{\partial X_n} \right)$, i.e. the Tjurina-algebra $T(f_1)$ coincides with the Milnor-algebra $M(f_1) = k[[X]]/(\frac{\partial f_1}{\partial X_0}, \ldots, \frac{\partial f_1}{\partial X_n})$, and in this case the result coincides with ([1], 12.6).

3. Results in the 1-semiquasihomogeneous case

Using a computer$^3$, from the theorem in section 2, we obtain easily:

<table>
<thead>
<tr>
<th>type</th>
<th>Tjurina-number</th>
<th>maximal set of linearly independent superdiagonal monomials</th>
<th>total number</th>
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<tr>
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<td></td>
<td></td>
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<tr>
<td>case 1: dimension $\equiv 1 \bmod 2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_0(0)$</td>
<td>16</td>
<td>$X_0X_1X_3$, $X_1X_2X_3$, $X_0X_2X_3$, $X_0X_1X_2X_3$</td>
<td>4</td>
</tr>
<tr>
<td>$E_6(t)$</td>
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<td>$X_1^2X_3$, $X_2^2X_3$, $X_1X_2X_3$, $X_2^3X_3$</td>
<td>4</td>
</tr>
<tr>
<td>$E_7$</td>
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<td>$\emptyset$</td>
<td>0</td>
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<tr>
<td>$E_8$</td>
<td>12</td>
<td>$X_0X_1^5$</td>
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<tr>
<td>$E_6(t)$</td>
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<td>0</td>
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<tr>
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<td>$E_{7,2}(t)$</td>
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</tr>
<tr>
<td>$E_8(t)$</td>
<td>10</td>
<td>$\emptyset$</td>
<td>0</td>
</tr>
</tbody>
</table>

$^3$collections done with REDUCE 3.5
Thus e.g. for \( n \) odd, the 1-sqh singularities with first term \( \hat{E}_8 \) (as in the theorem of section 1) are given by adding a constant multiple the monomial \( X_0 X_7^2 \). If the coefficient is not zero, an easy coordinate transformation leads to the only non quasihomogeneous 1-sqh singularity of that weight; it is given by the equation
\[
X_0(X_0 + X_1^2)(X_0 + tX_7^2) + X_0X_7^5 = 0 \quad (t \notin \{0,1\})
\]
with Tjurina number 11.

References


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