

Dee-Subproper Forcing

The concepts of d -proper and Dee-proper forcing were developed by Shelah in [PIF] and masterfully expounded by Uri Abraham in [PF]. In these notes we generalize the concepts to subproper forcing and then use them to solve an iteration problem left over from [PIF].

In §2 we introduce d -subproper forcing and prove that a revised countable support iteration of d -subproper forcings, subject to the usual restraints, yields an d -subproper forcing. d -proper forcings are trivially d -subproper. In addition, we show that every subcomplete forcing is d -subproper.

In §1 we lay the groundwork for §2 by revisiting the notion of subproper forcing. We slightly revise the definition and redo the proof of the iteration theorem, in preparation

for the more difficult proof in § 2.

In § 3 we introduce Dee -subproper forcing. This generalizes the notion of simple Dee -proper forcing, as defined in [PF], (A generalization of the broader concept is readily available but, alas, we were unable to prove a reasonable iteration theorem for it.) Subcomplete forcings are trivially Dee -subproper. We prove that an RCS iteration of forcings which are ω_1 -subproper and Dee -subproper, subject to the usual restraints, will not add new reals.

In § 4 we then apply our methods to a variant N' of Namba forcing. N' was treated extensively in [PIF] (where it is called Nm'). The conditions are Namba trees which have a special form:

A single finite stem followed by ω_2 many branchings at each node thereafter. An [PIF] it was shown that, assuming CH, \mathbb{N}' adds no reals and is essentially different from Namba forcing \mathbb{N} . It was also shown that \mathbb{N}' is semiproper if and only if Chang's conjecture holds. Shelah developed a theory of 'I-condition' forcings, which enabled him to iterate Namba forcing without adding new reals. Since, however, \mathbb{N}' does not appear to satisfy an I-condition, the question, whether \mathbb{N}' can be iterated without collapsing ω_1 was left open in [PIF]. In [SPSC] we showed, assuming CH + $2^{\omega_1} = \omega_2$, that \mathbb{N}' is subproper, hence can be iterated without collapsing ω_1 . In §4 we show, assuming only CH, that \mathbb{N}' is ω_1 -subproper and Dees-subproper.

Hence it can be iterated without adding new reals.

Bibliography

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* These notes are on my website.
(To find it, enter "Ronald B. Jensen"
in Google.)