III A Correction to §8 of [NFS]

The proof of Lemma 3.2 is circular. We first prove:

Lemma 3.2.1 Let M be a mouse. Let $\bar{M} = \mathrm{core}(M)$ with core map σ. Then $\sigma(p_{\bar{M}}) = p_M$ and $\sigma``P^*_{\bar{M}} \subset P^*_M$.

proof.

It suffices to show $\sigma(p_{\bar{M}}) = p_M$. Note that σ witnesses the goodness of $\langle M, \bar{M}, \rho \rangle$, where $\rho = \omega\rho^{\omega}_{\bar{M}}$. Coiterate $\langle M, \bar{M}, \rho \rangle$ against M, getting $\bar{\mathcal{J}} = \langle\langle \bar{M}_i \rangle, \ldots, \langle \bar{\pi}_{ij} \rangle, \bar{T} \rangle$ and $\mathcal{J} = \langle\langle M_i \rangle, \ldots, \langle \pi_{ij} \rangle, T \rangle$ of length θ.

Case 1 $\bar{M}_\theta$ is a proper segment of M_θ. Then $\bar{M}_\theta = \bar{M}$. But $\nu_0 > \rho$, since $\nu_0 \geq \rho$ and ρ is a cardinal in M. Hence, letting $A = A^{n, p_M}_M$ $(\omega\rho^n = \rho)$, we have $A \in \underline{\Sigma}^*(\bar{M})$; hence $A \in$ $\in J^{E^{M_\theta}}_{\nu_0} = J^{E^M}_{\nu_0} \subset M$, since ν_0 is a cardinal in M_θ. Contr!

Case 2 $\bar{M}_\theta = M_\theta$ is a nonsimple iterate of M.

Let $j+1$ be the maximal truncation point with $\eta = T(j+1)$. Then $M_j^* = \mathrm{core}(\bar{M}_\theta) = \bar{M}$, $M_j^* \in M_\eta$. But then $\bar{M}$ is a proper segment of M_η and the coiteration terminates at $\eta < \theta$. Contr!

Case 3 $\bar{M}_\theta = M_\theta$ is a simple iterate of M.

Then $p_{M_\theta} = \bar{\pi}_{0\theta}(p_{\bar{M}}) = \pi_{0\theta}(p_M)$. Since $\bar{M}$ is sound and $\bar{\pi}_{0\theta} \restriction \rho = \mathrm{id}$, we have $\bar{M} = \mathrm{core}(M_\theta)$ with core map $\bar{\pi}_{0\theta}$. But $\mathrm{crit}(\pi_{0\theta}) \geq \rho$, since otherwise $\omega\rho^\omega_{M_\theta} > \rho = \omega\rho^\omega_{\bar{M}_\theta}$. Contr!

Hence $\mathrm{rng}(\bar{\pi}_{0\theta}) \subset \mathrm{rng}(\pi_{0\theta})$, since each $x \in \bar{M}$ has the form $f(\xi, p_{\bar{M}})$ for a $\xi < \rho$, where $\bar{\pi}_{0\theta}(p_{\bar{M}}) = \pi_{0\theta}(p_M)$. Hence $\sigma = \pi_{0\theta}^{-1}\bar{\pi}_{0\theta}$ and $\sigma(p_{\bar{M}}) = p_M$.

<u>Lemma 3.2</u> Let M be a mouse. Let $\bar{M} = \text{core}_\alpha(M)$ with core map σ. Then $\sigma(p_{\bar{M}}) = p_M$ and $\sigma''P^*_{\bar{M}} \subset P^*_M$.

prf.

It again suffices to show: $\sigma(p_{\bar{M}}) = p_M$. Let $M' = \text{core}(M)$, σ' = the core map $\sigma' : M' \longrightarrow M$.

Set $\tilde{\sigma} = \sigma^{-1}\sigma' : M' \xrightarrow[\Sigma^*]{} \bar{M}$, ..

Then $\tilde{\sigma}(p_{M'}) = \sigma^{-1}(p_M) \in P^*_{\bar{M}}$.

Hence $p_{\bar{M}} \leq_* \tilde{\sigma}(p_{M'})$. By ind. on $\nu \in p_{M'}$ in descending order it follows that $\tilde{\sigma}(\nu) \in p_{\bar{M}}$ and $\tilde{\sigma}(W^\nu_{M'})$ is a generalized witness for $\tilde{\sigma}(\nu)$. Hence $\tilde{\sigma}(p_{M'}) = p_{\bar{M}}$.

$\sigma(p_{\bar{M}}) = \sigma\tilde{\sigma}(p_{M'}) = \sigma'(p_{M'}) = p_M$.

QED