Modulbeschreibung für Vertiefungsmodule des Wahlpflichtbereiches

Titel des Moduls	Optimale Stoppprobleme und ihre Anwendung in Statistik und Finanzmathematik
in englischer Sprache	Optimal stopping problems and their applications in statistics and financial mathematics

R	
A	X

	Vorlesung	Übung
Umfang	2	

Inhalt

Optimal stopping problems form an important and well-developed class of stochastic control problems. Their objective is to search for random times at which continuously observed random processes should be stopped, with the aim to optimise the expected values of given reward functionals. There are connections between optimal stopping problems for Markov processes and free-boundary problems for partial differential operators, which also appear in problems of mathematical physics. Such relationships lead to analytic expressions for the optimal expected values of the given functionals in particular models. Building on such connections, stochastic analysis theory can be used to establish that appropriate solutions of the free-boundary problems turn out to be solutions of the initial stopping problems. The course builds on the essentials of the theory of Markov processes (multi-dimensional diffusions and their infinitesimal generators) and contains basic concepts of optimal stopping problems and their connections to the free-boundary problems for partial differential operators. Some applications of the optimal stopping problems arising in mathematical statistics and finance are considered.

The problem of statistical sequential testing of two simple hypotheses about the dynamics of an observed stochastic process is to determine the time when the observations should be stopped and one of the hypotheses should be accepted. This time is sought to be optimal in the sense of minimal error probabilities and average observation time. The problem of disorder or on-line change-point detection is to find a time of alarm that should be sounded to indicate a change in probability characteristics of the observed process. This time is sought to be as close as possible to the unknown and unobservable (random) time of change in the sense of minimal probability of a false alarm and average time delay.

Contingent claims of American type are contracts that give their holder the right, but not the obligation, to exercise them at any time up to the maturity, and then receive a certain payoff from the issuer. The payoffs of American standard call and put options depend on the underlying risky asset value, taken at the time of exercise, while the payoffs of lookback (Russian) or integral (Asian) options depend on the running maximum or Lebesgue integral of the asset value, respectively. A natural mathematical problem, related to this type of contract, is to seek (stopping) times at which the holder should exercise the given claims on the underlying assets, in appropriate stochastic models of financial markets.

The lecture course will be organized as follows: optimal stopping problems for diffusion processes – free-boundary problems for partial differential operators – verification theorems – Bayesian and variational sequential hypotheses testing problems – Bayesian and variational disorder detection problems with linear and exponential delay penalties – pricing of perpetual American call and put options and their combinations (straddle and strangle) – pricing of American lookback (Russian) and integral (Asian) options (by means of the change-of-measure arguments). The course will also contain some elements of stochastic analysis (Brownian motion, diffusion processes, change-of-measure, filtering), mathematical statistics (hypotheses testing) and finance (no-arbitrage pricing).

Voraussetzungen	Stochastik I, Stochastik II.

Regelsemester	---

Abschluss	Prüfung

Prüfungszulassungsvoraussetzung	keine

Studienpunkte	4

R = Reine Mathematik
A = Angewandte Mathematik