1. Prove that the curve (affine variety) given by the equation \(xy = 1\) in \(\mathbb{A}^2\) is not isomorphic to \(\mathbb{A}^1\).

2. Show that the set \(X\) of points \(\{(t, t^2, t^3) : t \in k\} \subset \mathbb{A}^3\) is an irreducible affine variety and compute the ideal \(I(X) \subset k[x, y, z]\) by describing its generators. Is \(X\) isomorphic to \(\mathbb{A}^1\)?

3. Let \(X\) be the algebraic set in \(\mathbb{A}^3\) defined by the polynomials \(x^2 - yz\) and \(xz - x\). Show that \(X\) is the union of three irreducible components. Describe them and find their prime ideals.

4. Show that a \(k\)-algebra \(R\) is isomorphic to the affine coordinate ring of some algebraic set \(X \subset \mathbb{A}^n\) if and only if \(R\) is finitely generated as a \(k\)-algebra and has no nilpotent elements.

5. Show that any open subset of an irreducible topological space is dense and irreducible.

6. Compute the resultant of \(X^5 - 3X^4 - 2X^3 + 3X^2 + 7X + 6\) and \(X^4 + X^2 + 1\). Do these polynomials have a common factor in \(\mathbb{Q}[X]\)?

7. We fix \(X := \mathbb{C}\) and denote by \(\exp : \mathcal{O}_X \to \mathcal{O}^*_X\) the exponential map \(f \mapsto e^{2\pi i f}\). Prove that when \(U \subset X\) is simply connected then \(\exp_U\) is surjective. Give an example of an open subset \(U \subset X\) when this is no longer true. Describe the image sheaf of the morphism \(\exp\).

8. If \(f = a_nX^n + \cdots + a_0 \in K[X]\), where \(a_n \neq 0\), then the discriminant of \(f\) is defined as the quantity

\[
\text{disc}(f) := \frac{(-1)^{\binom{n}{2}}}{a_n} \text{Res}(f, f').
\]

Show that \(f\) has a multiple factor if and only if \(\text{disc}(f) = 0\).